



ELSEVIER

SCIENCE @ DIRECT®

PHYSICS LETTERS B

Physics Letters B 622 (2005) 349–355

www.elsevier.com/locate/physletb

Interacting viscous mixtures

Massimo Giovannini

*Centro “Enrico Fermi”, Compendio del Viminale, Via Panisperna 89/A, 00184 Rome, Italy
Department of Physics, Theory Division, CERN, 1211 Geneva 23, Switzerland*

Received 27 April 2005; accepted 6 July 2005

Available online 18 July 2005

Editor: L. Alvarez-Gaumé

Abstract

Gravitational and hydrodynamical perturbations are analysed in a relativistic plasma containing a mixture of interacting fluids characterized by a non-negligible bulk viscosity coefficient. The energy–momentum transfer between the cosmological fluids, as well as the fluctuations of the bulk viscosity coefficients, are analyzed simultaneously with the aim of deriving a generalized set of evolution equations for the entropy and curvature fluctuations. For typical length scales larger than the Hubble radius, the fluctuations of the bulk viscosity coefficients and of the decay rate provide source terms for the evolution of both the curvature and the entropy fluctuations. According to the functional dependence of the bulk viscosity coefficient on the energy densities of the fluids composing the system, the mixing of entropy and curvature perturbations is scrutinized both analytically and numerically.

© 2005 Elsevier B.V. Open access under [CC BY license](http://creativecommons.org/licenses/by/4.0/).

If a relativistic plasma contains a mixture of inviscid fluids with negligible transfer of energy and momentum, the evolution of entropy fluctuations is characterized, in the long-wavelength limit, by the absence of source terms containing curvature perturbations. This property has relevant consequences on the dynamics of the coupled system of gravitational and hydrodynamical perturbations. It implies, for instance, that curvature perturbations are conserved, in the long-wavelength limit, under rather general assumptions

[1–3]. Long-wavelength fluctuations in the spatial curvature determine, via the Sachs–Wolfe effect, the temperature inhomogeneities observed in the microwave sky (see, for instance, [4]).

Mixtures of relativistic fluids are a useful toy model that can be investigated with the purpose of inferring some general properties of the evolution equations of curvature and entropy fluctuations. Moreover, multifluid systems are per se relevant to the model-independent discussion of initial conditions of CMB anisotropy [1,4]: for instance, the five isocurvature modes supported by the predecoupling plasma may be discussed, in their simplest realization, by a truncated

E-mail address: massimo.giovannini@cern.ch (M. Giovannini).

Einstein–Boltzmann system of equations whose lower multipole moments reproduce indeed a multifluid hydrodynamical description [5].

One of the assumptions often invoked in the analysis of multifluid systems is that the bulk viscosity coefficient and its possible spatial variation have a negligible impact on the dynamics. While this assumption may be justified in some specific system, it may not be true in the early stages of the life of the Universe (see, for instance, [6–8]). Unlike other dissipative effects, the presence of bulk viscosity does not spoil the isotropy of the background geometry. Therefore, consider a mixture of two relativistic fluids (the a-fluid and the b-fluid) obeying a set of generally covariant evolution equations formed by the Einstein equations¹

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \frac{1}{2}T_\mu^\nu \quad (1)$$

and by the evolution equations of the energy–momentum tensors of each fluid of the mixture, i.e.,

$$\nabla_\mu T_a^{\mu\nu} = -\Gamma g^{\nu\beta} u_\beta (p_a + \rho_a), \quad (2)$$

$$\nabla_\mu T_b^{\mu\nu} = \Gamma g^{\nu\beta} u_\beta (p_a + \rho_a), \quad (3)$$

where u_β is the total velocity field of the mixture. Eqs. (2) and (3) describe the situation where the a-fluid decays into the b-fluid with decay rate Γ . It is evident from the form of Eqs. (2) and (3) that the total energy–momentum tensor of the mixture, i.e., $T^{\mu\nu} = T_a^{\mu\nu} + T_b^{\mu\nu}$ is covariantly conserved, i.e., $\nabla_\mu T^{\mu\nu} = 0$. The total energy–momentum tensor of each species is given by the sum of an inviscid contribution, denoted by $T_{a,b}^{\mu\nu}$ and by a viscous contribution, denoted by $\tilde{T}_{a,b}^{\mu\nu}$, i.e.,

$$T_{a,b}^{\mu\nu} = T_{a,b}^{\mu\nu} + \tilde{T}_{a,b}^{\mu\nu}, \quad (4)$$

$$T_{a,b}^{\mu\nu} = (p_{a,b} + \rho_{a,b})u_{a,b}^\mu u_{a,b}^\nu - p_{a,b}g^{\mu\nu}, \quad (5)$$

$$\tilde{T}_{a,b}^{\mu\nu} = \xi_{a,b}(g^{\mu\nu} - u_{a,b}^\mu u_{a,b}^\nu)\nabla_\alpha u_{a,b}^\alpha, \quad (6)$$

where the subscript in the various fluid quantities simply means that Eqs. (4), (5) and (6) hold, independently, for the a- and b-fluids. So, for instance, in

Eqs. (5) and (6), u_a^μ and u_b^μ denote the peculiar velocities of each fluid of the mixture.

In a spatially flat metric of Friedmann–Robertson–Walker (FRW) type characterized by a background line element

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)[d\tau^2 - d\vec{x}^2], \quad (7)$$

Eqs. (2) and (3) imply

$$\rho_a' + 3\mathcal{H}(\rho_a + \mathcal{P}_a) + a\bar{\Gamma}(\rho_a + p_a) = 0, \quad (8)$$

$$\rho_b' + 3\mathcal{H}(\rho_b + \mathcal{P}_b) - a\bar{\Gamma}(\rho_a + p_a) = 0, \quad (9)$$

where the prime denotes a derivation with respect to the conformal time coordinate τ and $\mathcal{H} = a'/a$. In Eqs. (8) and (9), $\mathcal{P}_{a,b}$ denote the total effective pressure of each species, i.e.,

$$\mathcal{P}_{a,b} = p_{a,b} - 3\frac{\mathcal{H}}{a}\bar{\xi}_{a,b}, \quad (10)$$

while $p_{a,b}$ denote the inviscid pressures of each species. In Eq. (10), $\bar{\xi}_{a,b}$ denote the bulk viscosity coefficient evaluated on the background geometry. As will be discussed later, the bulk viscosity coefficient may depend on both ρ_a and ρ_b . Eqs. (8) and (9) lead to the evolution of the total energy and pressure densities

$$\rho' + 3\mathcal{H}(\rho + \mathcal{P}) = 0, \quad (11)$$

where $\rho = \rho_a + \rho_b$ and $\mathcal{P} = \mathcal{P}_a + \mathcal{P}_b$. Eqs. (8) and (9) must be supplemented by the explicit background form of Eq. (1), i.e.,

$$3\mathcal{H}^2 = a^2\rho, \quad (12)$$

$$2(\mathcal{H}^2 - \mathcal{H}') = a^2(\rho + \mathcal{P}), \quad (13)$$

where, again, $(\rho + \mathcal{P})$ is the total effective enthalpy that contains the background viscosity coefficient of the mixture $\bar{\xi} = \bar{\xi}_a + \bar{\xi}_b$.

We are now interested in deriving the evolution of the entropy and total-curvature fluctuations of the system. Both the entropy perturbations and the perturbations in the total spatial curvature can be written in terms of ζ_a and ζ_b which are related in the off-diagonal gauge [9] (see also [10,11]) to the density contrasts of the individual fluids of the mixture [12]:

$$\mathcal{S} = -3(\zeta_a - \zeta_b), \quad (14)$$

$$\zeta = \frac{\rho_a'}{\rho'}\zeta_a + \frac{\rho_b'}{\rho'}\zeta_b. \quad (15)$$

¹ Units of $8\pi G = 1$ will be used throughout. Notice, to avoid confusions, that the Latin (lower-case roman) subscripts a, b, c, d, ... will denote, in the present Letter, different fluids present in the relativistic plasma. Greek (lower-case) subscripts will denote tensor indices. Latin (lower-case italic) subscripts i, j, k, \dots will denote the spatial components of a tensor.

In the following we are going to exploit the off-diagonal gauge [9] (also called uniform-curvature gauge [10]), which is particularly convenient for the problem at hand. The results will be exactly the same of those obtainable in the framework of gauge-independent descriptions (see [13]). In fact, the quantities \mathcal{S} and ζ , defined in terms of ζ_a and ζ_b , are invariant under infinitesimal coordinate transformations. Consequently they can be computed in any suitable (non-singular) coordinate system.

Concerning Eq. (15) it could be noticed that the gauge-invariant definition of the spatial curvature perturbation can be slightly different for wavelengths smaller than the Hubble radius. However, since in the problem at hand we are mainly interested in super-Hubble fluctuations, Eq. (15) is numerically equivalent to the curvature inhomogeneities defined, more conventionally, from the curvature fluctuations on comoving orthogonal hypersurfaces [2,14,15] (see also [4]). The physical interpretation of the entropy fluctuations defined in Eq. (14) can be understood, for instance, in the case of an inviscid mixture of cold dark matter (CDM) and radiation. In this situation the a-fluid is given by pressureless matter and the b-fluid by radiation. Applying the covariant conservation equations for two (inviscid and non-interacting) species, it is clear that \mathcal{S} is nothing but the fractional fluctuation in the specific entropy $\varsigma = T^3/n_{\text{CDM}}$ (where T is the radiation temperature and n_{CDM} is the CDM number density), i.e., the entropy density per CDM particle

$$\mathcal{S} \equiv \frac{\delta \varsigma}{\varsigma} = \delta_{\text{CDM}} - \frac{3}{4} \delta_r, \quad (16)$$

where δ_{CDM} and δ_r are the density contrasts for the CDM and for the radiation fluids.

In the off-diagonal gauge the spatial components of the perturbed metric vanish and, hence, the only components of the perturbed line element are:

$$\delta g_{00} = 2a^2 \phi, \quad \delta g_{0i} = -a^2 \partial_i B. \quad (17)$$

Since, in the long-wavelength limit, $(\mathcal{H}' - \mathcal{H}^2)\phi = \mathcal{H}^2 \zeta$, it turns out that in the off-diagonal gauge, δg_{00} is connected to ζ . As anticipated, ζ_a and ζ_b can be expressed in terms of the fluctuations of the density contrasts of the individual fluids, i.e.,

$$\zeta_a = -\frac{\mathcal{H}\rho_a}{\rho'_a} \delta_a, \quad \zeta_b = -\frac{\mathcal{H}\rho_b}{\rho'_b} \delta_b. \quad (18)$$

The evolution equations obeyed by the density contrasts $\delta_a = \delta\rho_a/\rho_a$ and $\delta_b = \delta\rho_b/\rho_b$ are derived by perturbing Eqs. (2) and (3) to first order in the amplitude of the metric and hydrodynamical fluctuations:

$$\begin{aligned} \delta'_a + (3\mathcal{H} + a\bar{\Gamma})(c_{sa}^2 - w_a)\delta_a \\ + \frac{9\mathcal{H}^2}{a\rho_a} [\bar{\xi}_a(\phi + \delta_a) - \delta\xi_a] \\ + a(1 + w_a)\bar{\Gamma}(\delta_r + \phi) \\ + \left[(1 + w_a) - 6\frac{\mathcal{H}\bar{\xi}_a}{a\rho_a} \right] \theta_a = 0, \end{aligned} \quad (19)$$

$$\begin{aligned} \delta'_b + 3\mathcal{H}(c_{sb}^2 - w_b)\delta_b \\ + a\bar{\Gamma}\frac{\rho_a}{\rho_b} [(1 + w_a)(\delta_b - \delta_r - \phi) - (1 + c_{sa}^2)\delta_a] \\ + \frac{9\mathcal{H}^2}{a\rho_b} [\bar{\xi}_b(\phi + \delta_b) - \delta\xi_b] \\ + \left[(1 + w_b) - 6\frac{\mathcal{H}\bar{\xi}_b}{a\rho_b} \right] \theta_b = 0. \end{aligned} \quad (20)$$

Concerning Eqs. (19) and (20) a few comments are in order:

- for notational convenience the barotropic indices (i.e., w_a , w_b) and the sound speeds (i.e. c_{sa}^2 and c_{sb}^2) have been introduced for the inviscid component of each species of the plasma; if the inviscid component is parametrized in terms of a perfect relativistic fluid $c_{sa,b}^2 \equiv w_{a,b}$;
- $\delta_r = \delta\Gamma/\bar{\Gamma}$ is the fractional fluctuation of the decay rate computed in the off-diagonal gauge;
- $\delta\xi_a$ and $\delta\xi_b$ denote the fluctuations of the bulk viscosity coefficients; later on it will also be convenient to introduce the fluctuation in the total viscosity, i.e., $\delta\xi = \delta\xi_a + \delta\xi_b$;
- finally, $\theta_a = \partial_i v_a^i = \nabla^2 v_a$ and $\theta_b = \partial_i v_b^i = \nabla^2 v_b$ are the divergences of the peculiar velocity field of each species; note that the global velocity $\theta = \partial_i v^i$ field (with $\delta T_0^i = (\rho + \mathcal{P})v^i$) is recovered from θ_a and θ_b by recalling that $(\rho + \mathcal{P})\theta = (\rho_a + \mathcal{P}_a)\theta_a + (\rho_b + \mathcal{P}_b)\theta_b$.

Eqs. (19) and (20) must be supplemented by the perturbed components of Eq. (1); in particular by the Hamiltonian and momentum constraints

$$\mathcal{H}\nabla^2 B + 3\mathcal{H}^2 \phi + \frac{a^2}{2} \delta\rho = 0, \quad (21)$$

$$\nabla^2[\mathcal{H}\phi + (\mathcal{H}^2 - \mathcal{H}')B] + \frac{a^2}{2}(\rho + \mathcal{P})\theta = 0, \quad (22)$$

and by the other two equations stemming from the spatial components (i.e., $(i = j)$ and $(i \neq j)$) of Eq. (1)

$$\phi' + \left(\mathcal{H} + 2\frac{\mathcal{H}'}{\mathcal{H}}\right) - \frac{a^2}{2\mathcal{H}}\left[\delta p - 3\frac{\mathcal{H}}{a}\delta\xi - \frac{\bar{\xi}}{a}(\theta - 3\mathcal{H}\phi)\right] = 0, \quad (23)$$

$$B' + 2\mathcal{H}B + \phi = 0. \quad (24)$$

In Eq. (21) the global energy and pressure density fluctuations (i.e., $\delta\rho$ and δp) have been introduced. As is clear from Eqs. (21)–(24), one of the advantages of the off-diagonal formulation is the absence of second time derivatives of the metric fluctuations. Strictly speaking the evolution equations of θ_a and θ_b should be added to the system. However, they are only relevant for typical length scales smaller than the Hubble radius at a given time. Since we are interested in the opposite regime, these equations will be omitted, but they will be discussed elsewhere in their full generality (see [13]).

By combining Eqs. (21) and (23) the evolution equation for ζ can be easily obtained; it is given by

$$\dot{\zeta} = -\frac{3}{2}\frac{H}{\bar{H}}\left[(\dot{\xi}_a + \dot{\xi}_b)\zeta + H(\delta\xi_a + \delta\xi_b)\right] + \frac{\dot{\rho}_a}{2\bar{H}}(c_{sb}^2 - c_{sa}^2)(\zeta_a - \zeta), \quad (25)$$

where we passed, for later convenience, from the conformal time coordinate τ to the cosmic time coordinate t (i.e., $dt = a(\tau) d\tau$).

Eqs. (19) and (20) lead to the evolution equations of ζ_a and ζ_b whose explicit form is given by

$$\dot{\zeta}_a + \left[\frac{\dot{q}_a}{q_a} + (3H + \bar{\Gamma})(1 + c_{sa}^2)\right]\zeta_a + \frac{9}{q_a}\left[H^2\delta\xi_a - \bar{\xi}_a\dot{H}\zeta\right] = \frac{p_a + \rho_a}{q_a}\bar{\Gamma}\left[\delta\Gamma + \frac{\dot{H}}{H^2}\zeta\right], \quad (26)$$

and by

$$\dot{\zeta}_b + \left[\frac{\dot{q}_b}{q_b} + 3H(1 + c_{sb}^2)\right]\zeta_b - \bar{\Gamma}\frac{\dot{q}_a}{q_b}(1 + c_{sa}^2)\zeta_a + \frac{9}{q_b}\left[H^2\delta\xi_b - \dot{H}\bar{\xi}_b\zeta\right] = -\frac{p_a + \rho_a}{q_b}\bar{\Gamma}\left[\delta\Gamma + \frac{\dot{H}}{H^2}\zeta\right], \quad (27)$$

where

$$q_a = \frac{\dot{\rho}_a}{H}, \quad q_b = \frac{\dot{\rho}_b}{H}. \quad (28)$$

Various identities can be used to bring Eqs. (25), (26) and (27) to slightly different (but equivalent) forms. In particular:

- using Eq. (15), we can always trade the combinations $(\zeta_a - \zeta)$ and $(\zeta_b - \zeta)$ for $\dot{\rho}_a/\dot{\rho}(\zeta_a - \zeta_b)$ and $\dot{\rho}_b/\dot{\rho}(\zeta_a - \zeta_b)$;
- according to Eq. (14), $(\zeta_a - \zeta_b) = -\mathcal{S}/3$;
- by virtue of the background equations (12) and (13), $\dot{H}/H = \dot{\rho}/(2\rho)$;
- if the inviscid component of each fluid of the mixture is a perfect fluid, then $c_{s,a}^2 = w_a$ and $c_{s,b}^2 = w_b$;
- finally the background evolution of each fluid, i.e., Eqs. (8) and (9), may always be employed to obtain equivalent forms of the above equations.

Specific limits of Eqs. (25)–(27) will now be reproduced. In the limit $\bar{\xi}_a = \bar{\xi}_b = 0$, with $\delta\Gamma = \delta\xi_a = \delta\xi_b = 0$ and $\dot{\Gamma} = 0$, Eqs. (25)–(27) read

$$\dot{\zeta} = -\frac{H\dot{\rho}_a\dot{\rho}_b}{\dot{\rho}^2}(w_b - w_a)\mathcal{S}, \quad (29)$$

$$\dot{\zeta}_a = \frac{\bar{\Gamma}}{6}(w_a + 1)\frac{\dot{\rho}_b\rho_a}{\dot{\rho}_a}\mathcal{S}, \quad (30)$$

$$\dot{\zeta}_b = 0. \quad (31)$$

In the case $w_a = 0$ and $w_b = 1/3$, Eqs. (29)–(31) coincide with the set of equations used in Ref. [16] to describe the radiative decay of a massive curvaton whose effective pressure, at the oscillatory stage, reproduces that of dusty matter, i.e., $w_a = 0$. It is then clear, taking the difference between Eqs. (31) and (30), that the evolution equation of entropy perturbations

$$\dot{\mathcal{S}} = \frac{\bar{\Gamma}}{2}(w_a + 1)\frac{\dot{\rho}_a\rho_a}{\rho\dot{\rho}_b}\left(1 - \frac{\dot{\rho}_b^2}{\dot{\rho}_a^2} - 2\frac{\rho}{\rho_a}\right)\mathcal{S} \quad (32)$$

is homogeneous and does not contain any ζ -dependent source term.

Sticking to the case of the radiative decay of a dusty fluid, but including the fluctuations of the decay rate, the following system of evolution equations

$$\dot{\zeta} = \frac{\dot{\rho}}{6\bar{H}}(\zeta_a - \zeta), \quad (33)$$

$$\dot{\zeta}_a - \frac{\dot{g}_a}{g_a} \zeta_a = -g_a \left(\delta_\Gamma + \frac{\dot{H}}{H^2} \zeta \right), \quad (34)$$

can be derived from Eqs. (25)–(27) when $w_b = 1/3$ and $w_a = 0$. In Eqs. (33) and (34), $g_a = -H\rho_a/\dot{\rho}_a$. Eqs. (33) and (34) describe the situation discussed in Ref. [17], where the dynamics of the inflaton with inhomogeneous decay rate has been discussed (see, for instance, also [18–21]). If the spatial fluctuations of the decay rate are not a function of the local energy density of the mixture, curvature fluctuations may be generated for length scales larger than the Hubble radius.

Consider now the case where the $\bar{\Gamma}$ is constant, the decay is homogeneous (i.e., $\delta_\Gamma = 0$), but $\xi_a = \xi_a(\rho_a)$ and $\xi_b = \xi_b(\rho_b)$. This occurrence implies that

$$\delta \xi_a = -\frac{\dot{\xi}_a}{H} \zeta_a, \quad \delta \xi_b = -\frac{\dot{\xi}_b}{H} \zeta_b. \quad (35)$$

Hence, from Eqs. (25)–(27), we obtain, respectively

$$\dot{\zeta} = -\frac{\dot{\rho}_b}{\rho} \left[\frac{H \dot{\rho}_a}{\dot{\rho}} (w_b - w_a) + \frac{\dot{\rho}}{4\rho} \left(\dot{\xi}_a - \frac{\dot{\rho}_a}{\dot{\rho}_b} \dot{\xi}_b \right) \right] \mathcal{S}, \quad (36)$$

$$\dot{\zeta}_a = \frac{\dot{\rho}_b}{6\rho \dot{\rho}_a} [\bar{\Gamma}(w_a + 1)\rho_a + 9H^2 \bar{\xi}_a] \mathcal{S}, \quad (37)$$

$$\dot{\zeta}_b = -\frac{\dot{\rho}_a}{3\dot{\rho}_b} \left[\bar{\Gamma}(w_a + 1) \left(1 - \frac{\rho_a}{2\rho} \right) + \frac{3}{2} \bar{\xi}_b \right] \mathcal{S}. \quad (38)$$

Again, in this case, it can be easily argued that the evolution of entropy fluctuations obeys a homogeneous equation in \mathcal{S} . In fact, combining Eqs. (37) and (38) it is possible to obtain

$$\dot{\mathcal{S}} = - \left[\frac{\dot{\rho}_a \rho_a}{2\rho \dot{\rho}_b} \bar{\Gamma}(w_a + 1) \left(1 - \frac{\dot{\rho}_b^2}{\dot{\rho}_a^2} - 2 \frac{\rho}{\rho_a} \right) + \frac{3}{2} \left(\bar{\xi}_a + \frac{\dot{\rho}_b^2}{\dot{\rho}_a^2} \bar{\xi}_b \right) \right] \mathcal{S}. \quad (39)$$

This conclusion can be, however, evaded if ξ_a and ξ_b are functions both of ρ_a and ρ_b , i.e., $\xi_a = \xi_a(\rho_a, \rho_b)$ and $\xi_b = \xi_b(\rho_a, \rho_b)$. In this case

$$\delta \xi_a = -\frac{\dot{\xi}_a}{H} (\zeta_a + \zeta_b), \quad \delta \xi_b = -\frac{\dot{\xi}_b}{H} (\zeta_a + \zeta_b). \quad (40)$$

Thus, in the situation described by Eq. (40), Eq. (39) will inherit two extra terms at the right-hand side, i.e.,

$$-\frac{9H^2}{\dot{\rho}_b \dot{\rho}_a} (\dot{\rho}_a \dot{\xi}_b \zeta_a - \dot{\rho}_b \dot{\xi}_a \zeta_b), \quad (41)$$

which cannot be recast, for generic ξ_a and ξ_b , in a single term proportional to \mathcal{S} .

A relevant issue to be addressed concerns the phenomenological viability of interacting viscous mixtures. Consider, for instance, a model where the decay rate is constant but inhomogeneous (i.e., $\delta_\Gamma \neq 0$) and $\xi_a = \epsilon \sqrt{\rho_a}$ (where ϵ is constant). The viscosity coefficient of the b-fluid vanishes, i.e., $\xi_b = 0$. This model describes the situation where the a-fluid is initially dominant and characterized by a viscosity proportional to ϵ . Furthermore, if we want the universe to be expanding, we must also require $\epsilon < (w_a + 1)/\sqrt{3}$. The a-fluid will start its decay for a typical cosmic time $t_\Gamma \sim \bar{\Gamma}^{-1}$, and then the background will be dominated by the b-fluid while the energy density of the a-fluid, i.e., ρ_a , will decay exponentially. Also the background viscosity will decay exponentially, since $\bar{\xi}_a = \epsilon \sqrt{\rho_a}$. These aspects are illustrated in Fig. 1 (plot at the left-hand side) where, for two different values of ϵ , the common logarithm (i.e., the logarithm to base 10) of ρ_a and ρ_b are reported. From the point of view of the background, this model is perfectly viable and it leads to a final stage of expansion dominated by the b-fluid. To make the example even more explicit, one can think of the situation where the a-fluid is given by dust (i.e., $w_a = 0$) or stiff matter (i.e., $w_a = 1$). The b-fluid may be taken, for instance, to coincide with radiation (i.e., $w_b = 1/3$).

The dynamics of curvature fluctuations may be described, for practical reasons, by expressing the evolution equations in terms of ζ and δ_a , i.e., the curvature fluctuations and the density contrast of the a-fluid. Given the relations (15) and (18) ζ_b , ζ_a and δ_b can always be obtained as linear combinations (with background-dependent coefficients) of ζ and δ_a . From Eqs. (19) and (25) the relevant evolution equations can be written as

$$\dot{\zeta} = -\frac{1}{4\bar{H}} \left[3 \frac{H\epsilon}{\sqrt{\rho_a}} + 2(w_b - w_a) \right] \times (\dot{\rho}_a \zeta + H\rho_a \delta_a), \quad (42)$$

$$\begin{aligned} \dot{\delta}_a + \frac{9\epsilon H^2}{2\sqrt{\rho_a}} \delta_a + \frac{\dot{H}}{H^2} \left[\frac{9\epsilon H^2}{\sqrt{\rho_a}} + \bar{\Gamma}(w_a + 1) \right] \zeta \\ = -\bar{\Gamma}(w_a + 1) \delta_\Gamma. \end{aligned} \quad (43)$$

Eqs. (42) and (43) describe the evolution of ζ and δ_a for typical wavelengths larger than the Hubble radius. Initial conditions of the system are then set by

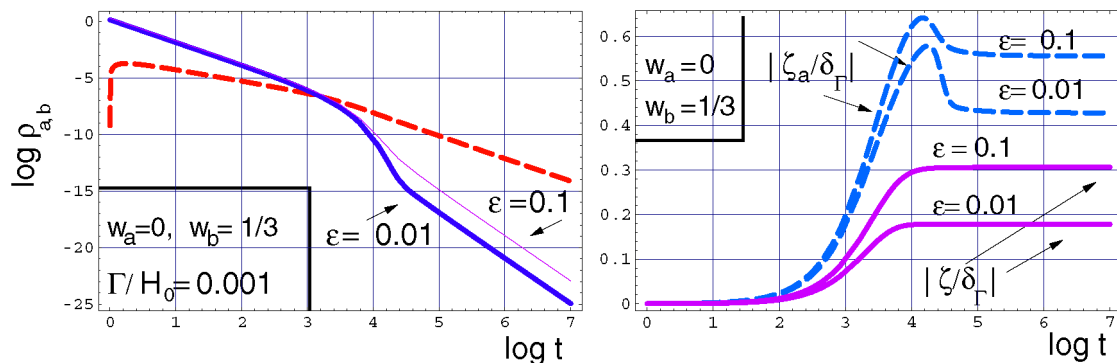


Fig. 1. The evolution of the background (left-hand plot) and of the fluctuations (right-hand plot) is illustrated. The parameters of the mixture are fixed in such a way that $w_a = 0$, $w_b = 1/3$, $\Gamma/H_0 \sim 10^{-3}$, $\xi_a = \epsilon/\sqrt{\rho_a}$, $\xi_b = 0$; H_0 denotes the value of the Hubble parameter at the initial integration time. In the left plot, the dashed curve represents the evolution of the energy density of the decay products (radiation) while the full lines represent the evolution of the energy density of the decaying component for different values of ϵ . In the right-hand side plot the dashed curves illustrate the behaviour of $|\zeta_a|$ while the full lines represent the evolution of $|\zeta|$. Both $|\zeta|$ and $|\zeta_a|$ are given in units of δ_Γ .

requiring $\zeta(t_0) = 0$ and $\delta_a(t_0) = \delta_b = 0$, where t_0 is the initial integration time. From Fig. 1 (plot at the right-hand side) the evolution is such that curvature fluctuations grow from 0 to a value proportional to δ_Γ , i.e., proportional to the fluctuations of the decay rate over length scales larger than the Hubble radius. The final asymptotic value of ζ can be determined analytically and it turns out to be

$$|\zeta_{\text{final}}| \simeq \frac{1}{6} \left(\frac{1 + 3\sqrt{3}\epsilon}{1 - \sqrt{3}\epsilon} \right). \quad (44)$$

In the limit $\epsilon \rightarrow 0$, the results reproduce the findings of Ref. [17] leading to a Bardeen potential $|\Psi_{\text{final}}| \simeq \delta_\Gamma/9$, which implies $|\zeta_{\text{final}}| \simeq \delta_\Gamma/6$ by using the well-known relation of ζ and Ψ in a radiation-dominated phase (see, for instance, [4]). In the example discussed so far the values of δ_Γ have been taken in the ranges 10^{-6} – 10^{-9} .

The class of examples reported so far can be generalized in various ways. Different barotropic indices for the fluids of the mixture can be studied. Eq. (44) can then be generalized to the cases of generic w_a and w_b . Furthermore, the functional dependence of the viscosity coefficients can be chosen to be different. Possible generalizations will be present elsewhere [13]. We would like to point out that the simple examples presented here may be made more realistic by thinking that a dust fluid is an effective description of a scalar field oscillating in a quadratic potential [22]. Thus, the simple fluid model of a dust fluid decaying into radi-

ation has been used [17] (with some caveats [20]) to infer some properties of the inflaton decay when the inflaton decay rate is not homogeneous. If the inhomogeneous decay occurs after an inflationary phase at low curvature (i.e., $H_{\text{inf}} \ll 10^{-6} M_{\text{P}}$), it is plausible to argue that the spectrum of δ_Γ may be converted into the spectrum of ζ for typical frequencies smaller than the Hubble rate. We are not interested here in supporting a specific model of inhomogeneous reheating. The purpose of the examples discussed so far is purely illustrative. However, the lesson to be drawn is that bulk viscous stresses may play a relevant role.

In the present Letter, various results have been achieved. First of all, the concept of interacting viscous mixtures has been introduced, i.e., a mixture of interacting fluids with viscous corrections. In this framework, the coupled evolution of curvature and entropy fluctuations has been derived in the case where both the decay rate and the bulk viscosity coefficients are allowed to fluctuate over typical length scales larger than the Hubble radius. Different situations have been systematically discussed. If the decay rate is constant and homogeneous, with bulk viscosities that depend separately on the energy density of each fluid of the mixture, the evolution of entropy fluctuations obeys a source-free evolution equation. If, on the contrary, the bulk viscosity has a more general dependence on the energy densities of the fluids composing the mixture, the evolution equations of the entropy perturbations may inherit a source term that involves, in

one way or another, curvature fluctuations. In similar terms, if the decay rate is allowed to fluctuate without being a function of the local density of the fluid, entropy fluctuations will not obey a source-free equation.

References

- [1] H. Kodama, M. Sasaki, *Prog. Theor. Phys. Suppl.* 78 (1984) 1.
- [2] J. Bardeen, P. Steinhardt, M. Turner, *Phys. Rev. D* 28 (1983) 679.
- [3] S. Weinberg, *Phys. Rev. D* 67 (2003) 123504.
- [4] M. Giovannini, Theoretical tools for CMB physics, CERN-PH-TH/2004-140, astro-ph/0412601.
- [5] M. Bucher, K. Moodley, N. Turok, *Phys. Rev. D* 62 (2000) 083508.
- [6] J.D. Barrow, *Nucl. Phys. B* 310 (1988) 743.
- [7] J. Barrow, *Phys. Lett. B* 180 (1986) 335.
- [8] J. Barrow, *Phys. Lett. B* 187 (1987) 12.
- [9] R. Brustein, M. Gasperini, M. Giovannini, V.F. Mukhanov, G. Veneziano, *Phys. Rev. D* 51 (1995) 6744.
- [10] J. Hwang, *Astrophys. J.* 375 (1990) 443.
- [11] J.c. Hwang, H. Noh, *Class. Quantum Grav.* 19 (2002) 527.
- [12] K.A. Malik, D. Wands, *JCAP* 0502 (2005) 007.
- [13] M. Giovannini, Imperfect cosmological perturbations, in preparation.
- [14] R. Brandenberger, R. Kahn, W. Press, *Phys. Rev. D* 28 (1983) 1809.
- [15] D.H. Lyth, *Phys. Rev. D* 31 (1985) 1792.
- [16] K.A. Malik, D. Wands, C. Ungarelli, *Phys. Rev. D* 67 (2003) 063516.
- [17] G. Dvali, A. Gruzinov, M. Zaldarriaga, *Phys. Rev. D* 69 (2004) 023505.
- [18] L. Kofman, astro-ph/0303614.
- [19] M. Postma, *JCAP* 0403 (2004) 006.
- [20] A. Mazumdar, M. Postma, *Phys. Lett. B* 573 (2003) 5;
A. Mazumdar, M. Postma, *Phys. Lett. B* 585 (2004) 307, Erratum.
- [21] R. Allahverdi, *Phys. Rev. D* 70 (2004) 043507.
- [22] M.S. Turner, *Phys. Rev. D* 28 (1983) 1243.